Institute of Actuaries of India

ACET September 2019 Solutions

Mathematics

1. C. $1.23425001 + 2.03049912 = 3.26474913$. After rounding this number 4 places after decimal, we obtain 3.2647.

2. A. The determinant of the matrix is easily computed as $b(b-2)(b+1)$. When $b \neq -1,0,2$, this determinant is non-zero. Hence rank of the given matrix is 3.

3. C. The system has no solution, as the lines are parallel, i.e., they never intersect.

4. B. Given limit is equivalent to

$$\lim_{x\to 1/2} \frac{t^2}{x-\frac{1}{2}}\Bigg|_{t=2}^{t=g(x)} = \lim_{x\to 1/2}\frac{g^2(x)-4}{x-\frac{1}{2}} = \lim_{x\to 1/2}\frac{g(x)-2}{x-\frac{1}{2}} \times \lim_{x\to 1/2}(g(x)+2)$$

$$= \lim_{x\to 1/2}\frac{g(x)-g\left(\frac{1}{2}\right)}{x-\frac{1}{2}} \times \left[g\left(\frac{1}{2}\right)+2\right] = g'\left(\frac{1}{2}\right)\times(2+2) = 4g'\left(\frac{1}{2}\right).$$

5. D. Given limit $= \lim_{u\to\infty}\left(1+\frac{-3}{u}\right)^{u-1} = \lim_{u\to\infty}\left(1+\frac{-3}{u}\right)^{u} = e^{-3}$.

6. D. There is jump discontinuity at every integer value of x.

7. C. $\int_1^{1/e}\frac{\log_e y}{y+1}dy = \int_1^{e}\frac{\log_e z}{z(z+1)}dz$. Therefore,

$$h(e) = \int_1^e\left[\frac{\log_e y}{y+1}+\frac{\log_e y}{y(y+1)}\right]dy = \int_1^e\frac{\log_e y}{y}dy = \int_0^1 t\,dt = \frac{1}{2}.$$

Here $z=\frac{1}{y}$ and $\log_e y = t$.

8. C. $g\circ f:[0,\infty)\to(-\infty,\infty)$, $g\circ f(x) = g\big(f(x)\big) = g(x) = |x| = x, x\in[0,\infty)$. Hence $g\circ f$ is one-to-one. But $g:(-\infty,\infty)\to(-\infty,\infty)$ and $g(-3)=g(3)=3$. So, g is not one-to-one.

9. D. $4! = 24$ is divisible by 8. Hence $5!, 6!, 7!, \ldots, 99!$ are all divisible by 8.
Thus the required remainder is obtained when $1!+2!+3! = 9$ is divided by 8, which is 1.

10. B. $|x_n| = \left|2-\frac{1}{n}\right|$ has the least upper bound 2.

11. C. $f'(x)$ does not exist at $x=0$, since $\lim_{x\downarrow 0} f'(x) = -\infty$, $\lim_{x\uparrow 0} f'(x) = \infty$.

Moreover, $f(0)=1$ and $f(x)<1$, for all other x. Thus, $f(x)$ is maximum at $x=0$, although derivative does not exist there.

12. A. By the use of L'Hôpital's rule, the limit is reduced to $\lim_{x\to 0}\frac{2\cos 2x+\tau\cos x}{2x}$.
Since there is 0 in the denominator, for finiteness of the limit as $x\to 0$ through L'Hôpital's rule, we must have $\lim_{x\to 0}(2\cos 2x+\tau\cos x)=0$. Thus $2+\tau=0$, i.e., $\tau=-2$.

13. C. Here $f'(x)=\frac{\sin x\cos x}{|\sin x|}$, exists only when $x\neq n\pi, n=0,\pm 1,\pm 2,\ldots$..

14. A. Since the points 2 and 4 are closest to 3, the most suitable linear interpolation for approximating $f(3)$ is

$$\left(1-\frac{3-2}{4-2}\right)f(2)+\left(\frac{3-2}{4-2}\right)f(4)=\frac{1}{2}[f(2)+f(4)]=45.$$

15. C. The given equation is equivalent to

$$a\cos x-b\sin x=\sqrt{a^2+b^2}\cos(x+\theta)=d,$$

where $\cos\theta=a/\sqrt{a^2+b^2}$ and $\sin\theta=b/\sqrt{a^2+b^2}$. It simplifies to $\cos(x+\theta)=d/\sqrt{a^2+b^2}$, which has a solution if and only if $\left|\frac{d}{\sqrt{a^2+b^2}}\right|\leq 1$.

16. D. The condition $(\vec{a}\times\vec{b})=2(\vec{a}\times\vec{c})$ implies that the planes of the pairs of vectors $(\vec{a},\vec{b})$ and $(\vec{a},\vec{c})$ are identical, i.e., the three vectors $\vec{a},\vec{b}$ and $\vec{c}$ are co-planar. Therefore, the vector $\vec{b}\times\vec{c}$ is perpendicular to that plane, and in particular, to the vector $\vec{a}$.

17. B. For the rank to be equal to 1, all the columns should be a multiple of the first column. The second column is already a multiple. The only value of c that makes the third column a multiple also is -6.

18. A. Let $y=0.272727\ldots$, so $100y=27.272727\ldots=27+y$.
Thus $99y=27$, which implies $y=\frac{27}{99}=\frac{3}{11}$.
Also, $0.727272\ldots=1-y=\frac{8}{11}$.
As $y,x,(1-y)$ are H.P., we have $x=\frac{2y(1-y)}{y+(1-y)}=2y(1-y)=\frac{48}{121}$, a rational number lying between 0 and 1.

19. B. It is given that $\frac{\log_{10}b}{\log_{10}a}=\frac{\log_{10}c}{\log_{10}b}$. Hence, $\log_{10}a$, $\log_{10}b$ and $\log_{10}c$ are in G.P.

Statistics

20. D. If n be the number of sides, then $\binom{n}{2} - n = 65 \Rightarrow n = 13$.

21. D. ${}^7P_3 \times 4 \times 3! = 7!$.

22. A. Alphabetical order of the letters A, K, N, R. Number of words starting with A is 6 and same is true for K and N. The first word starting with R is $RAKN$, next word will be $RANK$. Thus the position of the word $RANK$ is $(6 \times 3 + 1) + 1 = 20$.

23. C. A and B are already mutually exclusive. If they have to be independent too, then $P(A) \times P(B) = P(A \cap B) = 0,$ i.e., both $P(A)$ and $P(B)$ cannot be positive. Conversely, if $P(A) \neq 0$ and $P(B) \neq 0$, A and B cannot be independent.

24. D. $P(A^c) + P(B^c) = 2 - P(A) - P(B) = 2 - P(A \cup B) - P(A \cap B) = 1$.

25. B. A_1: coin drawn is two tailed
A_2: coin drawn is fair
B: the toss produces tail. Required probability $P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) = \frac{31}{42} \Rightarrow \frac{m}{2m+1} \times 1 + \frac{m+1}{2m+1} \times \frac{1}{2} = \frac{31}{42} \Rightarrow m = 10$.

26. B. Arranging the 10 integers in ascending order without the median x:
2,3,3,7,9,11,13,17,19,21
6th position is the median position, and hence the only three integer options for the median are 9, 10 and 11.

27. D.

$$\sqrt{\frac{1}{5}\sum_{i=1}^{6}\left((ay_i + b) - \frac{1}{6}\sum_{j=1}^{6}(ay_j + b)\right)^2} = \sqrt{\frac{1}{5}\sum_{i=1}^{6}\left(ay_i - \frac{1}{6}\sum_{j=1}^{6} ay_j\right)^2}$$

$$= \sqrt{\frac{a^2}{5}\sum_{i=1}^{6}\left(y_i - \frac{1}{6}\sum_{j=1}^{6} y_j\right)^2} = |a|m.$$

28. C. Let A and B represent the two distributions. Then $\frac{\sigma_A}{\mu_A} = 0.4,\ \frac{\sigma_B}{\mu_B} = 0.5$.

$$\text{Therefore, } \sigma_A = 0.4\mu_A = 0.4 \times 25 = 10 \text{ and } \sigma_B = 0.5\mu_B = 0.5 \times 20 = 10,$$
$$\text{i.e., } \sigma_A - \sigma_B = 0.$$

29. A. The only two possible values of $|X - 2|$ are 0 and 1, and these occur with probabilities 0.3 and 0.7, respectively. Therefore, $E|X - 2| = 0.7$.

30. C.

$$P\left(x \le \frac{2}{3} \middle| x > \frac{1}{3}\right) = \frac{P\left(x > \frac{1}{3}, x \le \frac{2}{3}\right)}{P\left(x > \frac{1}{3}\right)} = \frac{\int_{1/3}^{2/3} cx^2 dx}{\int_{1/3}^{1} cx^2 dx} = \frac{\frac{c}{3}x^3\Big|_{1/3}^{2/3}}{\frac{c}{3}x^3\Big|_{1/3}^{1}} = \frac{\frac{8}{27} - \frac{1}{27}}{1 - \frac{1}{27}} = \frac{7}{26}.$$

31. A. $(x-50)/10 = 1.645$, i.e., $x = 50 + 1.645 \times 10 = 66.45$.

32. C. The probability of an single person being female and smoker is $0.6 \times 0.4 = 0.24$. The count of female smokers in a random sample of size 10 is a binomial random variable with $n = 10$ and $p = 0.24$. The probability of this count being 0 is 0.76^{10}.

33. C. The sum of two independent Poisson distributed random variables with mean 5 is another Poisson distributed random variable with mean 10. Therefore,

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - \frac{e^{-10}10^0}{0!} - \frac{e^{-10}10^1}{1!} = 1 - 11e^{-10}.$$

Alternative solution: Let X_1 and X_2 be the number of calls coming in the first and second minute, respectively. Therefore,

$$P(X_1 + X_2 = 0) = P(X_1 = 0)P(X_2 = 0) = \frac{e^{-5}5^0}{0!} \times \frac{e^{-5}5^0}{0!} = e^{-10};$$

$$P(X_1 + X_2 = 1) = P(X_1 = 1)P(X_2 = 0) + P(X_1 = 0)P(X_2 = 1) = 2 \times \frac{e^{-5}5^1}{1!} \times \frac{e^{-5}5^0}{0!}$$
$$= 10e^{-10};$$

$$P(X_1 + X_2 \geq 2) = 1 - P(X_1 + X_2 = 0) - P(X_1 + X_2 = 1) = 1 - 11e^{-10}.$$

34. A. Here, $f(x) = \frac{1}{2a}$ over $[a, 3a]$ and 0 elsewhere. Therefore,

$$E(X^2) = \int_a^{3a} \frac{1}{2a} x^2 dx = \frac{x^3}{6a}\bigg|_a^{3a} = \frac{27a^3 - a^3}{6a} = \frac{13a^2}{3}.$$

35. D. The given condition implies that the correlation between X and Y is -0.4. Therefore, the correlation between $5 - X$ and $2Y + 3$ is 0.4.

36. C. The expected price at Mumbai is $\mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(X - \mu_X) = 80 + 0.9 \times \frac{3.5}{2.5}(85 - 60) = 111.5$.

37. B. Number of all possible ways to drop four cards from the pack is $\binom{52}{4}$. Number of ways to drop one card from a particular suit is 13. Therefore, number of selections producing one card from each suit is 13^4.

38. B. Clearly, $S = \{2,3,5,7,11,13,17,19,23,29\}$ contains 10 elements. Thus both numerator and denominator can be selected in 10 ways. Hence required number is $10 \times 10 - 10 + 1 = 91$, as out of 10×10 selections, 10 numbers are just 1.

39. A. Using the first row sum and the second column sum, we have $P(Y = 1) = a + \frac{4}{9}$ and $P(X = 2) = a + \frac{1}{9}$. For independence, we must have

$$a = P(X = 2, Y = 1) = \left(a + \frac{4}{9}\right)\left(a + \frac{1}{9}\right)$$

which yields $a = \frac{2}{9}$. Sum over all probabilities gives $b + c = 1 - \frac{1}{3} - \frac{2}{9} - \frac{1}{9} - \frac{1}{9} = \frac{2}{9}$. Second row and first column sums give

$$b = P(X = 1, Y = 2) = \left(b + c + \frac{1}{9}\right)\left(b + \frac{1}{3}\right) = \frac{1}{3}\left(b + \frac{1}{3}\right), \qquad i.e.,\ b = \frac{1}{6}.$$

Hence, $c = \frac{2}{9} - \frac{1}{6} = \frac{1}{18}$.

Data Interpretation

40. A. Fruits and vegetables account for 33%, or about one-third of the total expenditure. If that amount is Rs. 2000, the total expenditure would be Rs. 6000 and the expenditure on Rice would be 25% of that, i.e., Rs. 1500.

41. C. $\frac{16\%}{84\%} \approx 1/5$.

42. D. Schools A and B both add up to 25.

43. D. The difference is 6 in both Schools C and E.

44. C. 2014, 2016 and 2017.

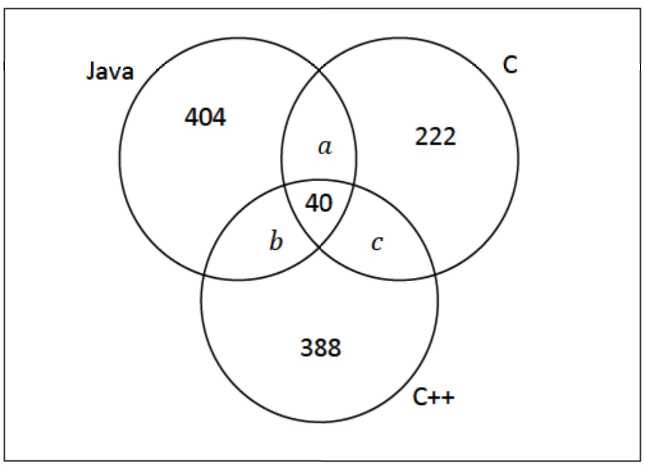
45. C. Smallest slope is visible between 2015 and 2016. Closer examination reveals that this period had smaller growth than the other period of small slope (2016 to 2017).

46. A. 1.2^2 time FY 2016-17 profits is 10 Crores. The answer is $1000/1.44 = 694.44$.

47. B. It is easy to visually identify that the ratio of the profit share to sales share is largest for Hyundai; total profits and sales do not matter.

48. B. The ratio is $0.17/0.13 \approx 1.307$.

49. D. The given information is shown in the Venn diagram. Additionally it is known that $a + 40 = 114$ and $b + c + 40 + 388 = 500$.



The number of students studying exactly two programming languages including C++ is $b + c = 500 - 40 - 388 = 72$.

50. B. The number of students studying exactly two languages is $a + b + c = (114 - 40) + 72 = 146$.

51. C. If the number of students studying the Java language is 568, then $b = 568 - 404 - 114 = 50$, i.e., $c = 72 - 50 = 22$. Therefore, the number of students studying the C language is $222 + 114 + c = 358$.

English

52. D
53. B
54. C
55. B
56. D
57. A
58. A
59. A
60. C
61. B
62. A

Logical Reasoning

63. A. A and E are husband and wife, with daughters-in-law D and F; H is a son of F. Therefore A is the grandfather of H.

64. C. Number of backward people who are educated = $11 + 3 = 14$.

65. C. N is the second play to be staged.

    O should be immediately followed by M, i.e., the order OM should be followed. N should be immediately followed by L, i.e., the order NL should be followed. N or O should not be the first or last play. Therefore, either of the orders NLOM or OMNL should be followed from Tuesday to Friday. One play is staged between K and L. So, the order is

Monday	K
Tuesday	N
Wednesday	L
Thursday	O
Friday	M

66. B. Leap years are generally separated by 4 years, the only exceptions being some years divisible by 100 (e.g., the year 1900). In such cases, the separation between consecutive leap years is 8 years. A longer separation is not possible.

67. B.

68. B. The relevant small cubes lie along the three edges common to the single corner (mentioned in the question). Adjusting for the triple counting of the corner cube, the requisite number is $3 \times 4 - 2 = 10$.

69. C. The gap between the hour hand and the minute hands reduces steadily starting from a gap of 45 minute spaces at 9 o'clock. The gap reduces at the rate of 55 minute spaces per hour (60 minutes). Therefore, they will converge in $\frac{60}{55} \times 45 = 49\frac{1}{11}$ minutes.

70. D.